UNIT 3: EXPRESSIONS AND EQUATIONS
WEEK 12

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* The Teacher Resource Binder includes student pages and reproducibles that are recommended for overhead transparencies.

WEEK 12 AT A GLANCE

Traditional Schedule (5 days@ 45+ minutes/day)

	Day 1	Day 2	Day 3	Day 4	Day 5
Classwork	12.2 (SP11-15)	12.3 (SP17-18)	12.1 (SP1-5)	12.1 (6-8)	Quiz Catch-up
Homework	SP0, 16	SP19-20	SP21-22	SP9-10	

Block Schedule (2 days@ 90+ minutes/day, 1 day@ 45+ minutes/day)

	Day 1	Day 2	Day 3
Classwork	12.2 (SP11-15) 12.3 (SP17)	12.1 (SP1-8)	Quiz Catch-up
Homework	SP0, 16, 18-19	SP9-10, 20-21	

CALIFORNIA MATHEMATICS CONTENT STANDARDS

Foundational Skills & Concepts

(Gr2) NS3.1	Use repeated addition, arrays, and counting by multiples to do multiplication.
(Gr4) AF1.5	Understand that an equation such as $y = 3x + 5$ is a prescription for determining a second number when a first number is given.
(Gr5) NS1.4	Determine the prime factors of all numbers through 50 and write the numbers as the product of their prime factors by using exponents to show multiples of a factor (e.g., $24 = 2 \times 2 \times 2 \times 3 = 2^3 \times 3$).
(Gr5) AF1.5	Solve problems involving linear functions with integer values; write the equation; and graph the resulting ordered pairs of integers on a grid.
(Gr6) NS2.2	Explain the meaning of multiplication and division of positive fractions and perform the calculations (e.g., $5/8 + 15/16 = 5/8 \times 16/15 = 2/3$).

Algebra Readiness

(Gr7) NS1.2	Add, subtract, multiply, and divide rational numbers (integers, fractions, and terminating decimals) and take positive rational numbers to whole-number powers.
(Gr7) MR2.0	Students use strategies, skills, and concepts in finding solutions.
(Gr7) MR2.2	Apply strategies and results from simpler problems to more complex problems.
(Gr7) MR3.2	Note the method of deriving the solution and demonstrate a conceptual understanding of the derivation by solving similar problems.

General Mathematics

(Gr4) MG2.1	Draw the points corresponding to linear relationships on graph paper (e.g.,, draw 10 points on the graph of the equation $y = 3x$ and connect them by using a straight line).
(Gr5) NS2.0	Students perform calculations and solve problems involving addition, subtraction, and simple multiplication and division of fractions and decimals.
(Gr6) NS2.1	Solve problems involving addition, subtraction, multiplication, and division of positive fractions and explain why a particular operation was used for a given situation.
(Gr7) AF1.5	Represent quantitative relationships graphically and interpret the meaning of a specific part of a graph in the situation represented by the graph.

INPUTS AND OUTPUTS 1

In this lesson, students determine the amount of time needed to save for the purchase of a camera, using input-output equations, tables, graphs, and words. Students learn the slope-intercept form of a line in a meaningful context, and they are led to appreciate the use of the formula.

This lesson occurs near the beginning of a cluster that focuses on linear functions. Within this cluster, students represent algebraic ideas visually, numerically, algebraically, and verbally (the fourfold way). In previous lessons, students used the fourfold way to represent geometric patterns. In future lessons, students will develop an informal understanding of equations of the form $y = mx + b$ in various contexts by making tables, writing equations, and sketching graphs that lead to linear relationships. Based on this work, concepts associated with linear functions will eventually be formalized.

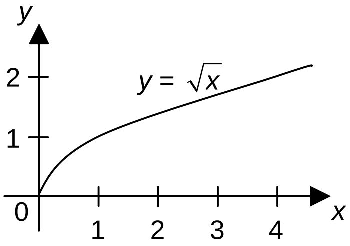
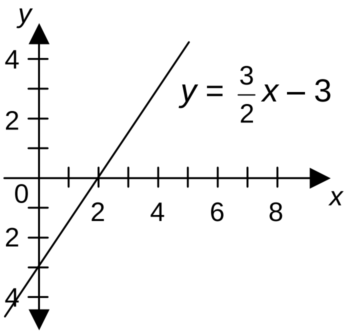
Math Goals (Standards for posting in **bold**)	• Use tables, graphs, equations, and words to solve problems. (Gr4 MG2.1; **Gr5 AF1.5**; Gr6 AF2.0; **Gr7 AF1.5**; Gr7 MR2.0) • Informally introduce the slope-intercept form of a line. (**Gr4 AF1.5**)
Summative Assessment	Future Week • Week 21: Direct Variation (Gr5 AF1.5) • Week 32: Slope (Gr4 AF1.5)

12.1 Inputs and Outputs 1

PLANNING INFORMATION

Estimated Time: 45 – 60 Minutes		
Student Pages * SP1: Ready, Set, Go * SP2: Saving for a Camera: Instructions * SP3: Saving for a Camera: Tables * SP4: Saving for a Camera: Graphs SP5: Saving for a Camera: Questions SP6: Saving for a Printer: Instructions and Tables SP7: Saving for a Printer: Graphs SP8: Saving for a Printer: Questions SP9: Brian’s Problem: Instructions and Tables SP10: Brian’s Problem: Graphs and Questions	**Materials** Colored pencils	**Reproducibles**
Homework SP6: Saving for a Printer: Instructions and Table SP7: Saving for a Printer: Graph SP8: Saving for a Printer: Questions	**Prepare Ahead**	**Management Reminders** Allow adequate time for summarizing. Students may need a lot of help making important connections between the different representations (numbers in the table; symbols in the equations; graphs; using vocabulary and expressing solutions properly).
Assessment * SP25: Knowledge Check 12 R52-53: Knowledge Challenge 12 A54: Weekly Quiz 12	**Strategies for English Learners** Students not yet speaking English can indicate answers to questions by pointing to evidence on the graph.	**Strategies for Special Learners** Link the formula for a linear function to a familiar context: $y = mx + b$ y = (money invested monthly)x $+$ (start amount in the bank)

* Recommended transparencies: See overheads 126-129 and 137 in the Teacher Resource Binder.

<table>
<tr><th colspan="2">THE WORD BANK</th></tr>
<tr><td>function</td><td>A function f on a domain D is a rule that assigns to each element x of D a unique value $y = f(x)$. (Read $f(x)$ as “f of x” or “the value of f at x”, NOT “f times x”.) Thus a function is an input-output rule that assigns to each input x a unique output $y = f(x)$.
Example: The function $y = mx + b$ assigns to each real number x the value $y = f(x) = mx + b$.</td></tr>
<tr><td>graph of a function</td><td>The graph of a function f on a domain D is the collection of points with coordinates (x, y), where x is in D and $y = f(x)$ is the value of the function at x.
Example: The graph of the function $y = mx + b$ is the straight line in the plane consisting of the pairs $(x, mx + b)$.
The graph of $f(x) = \sqrt{x}$, $x \geq 0$, is depicted below.

 </td></tr>
<tr><td>linear function</td><td>A linear function (in variables x and y) is a function that can be expressed in the form $y = mx + b$. The graph of $y = mx + b$ is a straight line with slope m and y-intercept b.
Example: The graph of the linear function $y = \frac{3}{2}x - 3$ is a straight line with slope $m = \frac{3}{2}$ and y-intercept $b = -3$.

 </td></tr>
</table>

MATH BACKGROUND

Functions and Relations	
Math Background 1 Preview/Warmup	There are two routes for developing the function idea. The route we have followed is to define a function as an input-output rule, and to define the graph of the function as ordered pairs of input and output values. Another more sophisticated route, which is common in school mathematics, is to define first a “relation” to be a set of ordered pairs, and then to define a function to be a relation with the “vertical line property. “ A *function*, according to this definition, is what we have defined as the *graph of the function*. The two routes lead to essentially the same class of objects. However, we have chosen the input-output definition because it is the definition used in college mathematics, particularly calculus. Further, the input-output definition is better adapted to understanding algebraic operations on functions and the operation of composition of functions.

12.1 Inputs and Outputs 1

PREVIEW / WARMUP

Whole Class

- ➢ SP1*
  Ready, Set, Go

Math Background 1

- Introduce the goals and standards for the lesson. Discuss important vocabulary as relevant.
- Students fill in the t-table, and discuss why the input/output equation should be $y = 3x + 5$.

INTRODUCE

Whole Class

- ➢ SP2*
  Saving for a Camera: Instructions
- ➢ SP3*
  Saving for a Camera: Table

- Preview the directions, and link the information to the equation.

  How much money must Julie save to purchase a digital camera? $240. ***Christina?*** $240

  If Julie deposited $100 in the bank, and 0 months have passed, what is her balance? Be sure students understand the meaning of account balance. After 0 months she will not have saved any extra, so she will still have $100. Using the equation and substituting 0 for x, show students that $10(0) + 100 = 100$.

- Do the same for another month or two.

  Do we have to compute the balance for months 1, 2, 3, 4, etc. to find the balance after month 3? No. If desired, we can use the formula to check **any** month. Students may find at this time that going one month at a time sequentially will be inefficient. However, allowing them to be inefficient now will help them recognize the usefulness of the formula.

  How much will Julie have after five months? $10(5) + 100 = 150$

  For how many months must Julie save until she has the $240? Verify using the equation. 14 months: $10(14) + 100 = 240$

- ➢ SP4*
  Saving for a Camera: Graph

Colored pencils

- Students make a graph (using a colored pencil) that shows how much Julie is saving each month.

  What is the coordinate on the graph that shows how much Julie has at the start? Graph the coordinate (0, 100). This is a good time to discuss appropriate labeling and scaling for the graph. Convention dictates that the number of months, the independent variable, will go on the horizontal (x) axis. The total amount saved in dollars, the dependent variable, will go on the vertical (y) axis.

  Do the coordinates form any pattern? They appear to be in a line—and they are. However, in this situation, the actual points are not connected because we are recording monthly data and fractions of months do not apply in this context. However, it is permissible to draw a trend line, solid or dashed, to show the linear relationship.

EXPLORE

Pairs

- SP3*
  Saving for a Camera: Table
- SP4*
  Saving for a Camera: Graph
- SP5
  Saving for a Camera: Questions

- Students use the equation to make a table and graph (with a different colored pencil) to show how much Christina is saving each month.

> Encourage students to use the formula rather than checking each month sequentially. The formula is an efficient way to find the desired solution, rather than a slower, iterative process.

- Students answer comprehension questions that link the context of the problem to the tables and graphs.

SUMMARIZE

Whole Class

- SP2*
  Saving for a Camera: Instructions
- SP3*
  Saving for a Camera: Table
- SP4*
  Saving for a Camera: Graph
- SP5
  Saving for a Camera: Questions

- Discuss solutions.

How can an equation be helpful in these problems? It is more efficient/quicker to find the solutions.

Who had the most money saved in the bank from the start? Julie. ***Was she the first to save $240?*** No. ***why not?*** Julie only saved $10 per month, so Christina eventually passed Julie.

Who had the least money saved in the bank from the start? Christina. ***Did she get to $240 last?*** No, Christina was first. ***Why?*** Christina saved more per month than Julie and eventually passed Julie.

Compare the amounts saved by each girl per month. Now compare the steepness of the graphed lines. Do you notice a pattern? Slower rate of savings translates into a flatter line. Faster rate of savings translates into a steeper line. The rate of change is called the slope of the line.

PRACTICE

Individuals/Pairs

- SP6
  Saving for a Printer: Instructions and Table
- SP7
  Saving for a Printer: Graph
- SP8
  Saving for a Printer: Questions

- Students represent another context (saving for a printer) using tables and graphs, and they answer questions about the problem using their mathematical representations. This is appropriate for homework.

EXTEND

Whole Class ➢ SP9 Brian's Problem: Instructions and Table ➢ SP10 Brian's Problem: Graph and Questions	• Challenge students to write an equation, make a table, and draw a graph to determine how long it will take Brian to save for the camera, the printer, and both if he has $100 in the bank and is going to save $20 each month. ***Why is ten months incorrect?*** Students may forget that to save for both, you will use the original $100 that was in the bank for the first item, and then for the second item it will take another seven months (i.e., you cannot use the $100 twice).

CLOSURE

Whole Class ➢ SP1* Ready, Set	• Review the goals and standards for the lesson.

SELECTED SOLUTIONS

<table>
<tr><td>SP1
Warmup</td><td colspan="2"><table><tr><th>x</th><th>y</th></tr><tr><td>10</td><td>35</td></tr><tr><td>1</td><td>8</td></tr><tr><td>0</td><td>5</td></tr><tr><td>9</td><td>32</td></tr><tr><td>11</td><td>38</td></tr><tr><td>20</td><td>65</td></tr></table>$y = 3x + 5$</td></tr>
<tr><td>SP5
Saving for a Camera: Questions</td><td>1. Julie ($100 > $40)
2. Christina ($25/month > $10/month); Christina's graph is steeper than Julie's; when comparing amount saved values in the tables, Christina's savings rise at a faster rate than Julie's.</td><td>3. At 4 months they both have $140. This is visible in the table. If 4 is substituted for x in each equation, they both have a y value of 140. The graphs intersect at the point (4, 140).
4. 14 months
5. 8 months
6. Christina</td></tr>
<tr><td>SP8
Saving for a Printer: Questions</td><td>1. Cary ($25 > $10)
2. Theresa ($20/month > $15/month); Theresa's graph is steeper than Cary's; when comparing amount saved values in tables, Theresa's savings rise at a faster rate than Cary's.</td><td>3. At 3 months they both have $70. This is visible in the table. If 3 is substituted in for x in each equation, they both have a y value of 70. The graphs intersect at (3, 70).
4. 7 months
5. 9 months (must round up)
6. Theresa</td></tr>
<tr><td>SP9
Brian's Problem: Instructions and Tables</td><td colspan="2">1. $y = 20x + 100$</td></tr>
<tr><td>SP10
Brian's Problem: Questions</td><td>1. Discuss graph.
2. 7 months</td><td>3. 3 months
4. 15 months</td></tr>
</table>

FRACTIONS: MULTIPLICATION 1

In this lesson students extend the concepts and procedures for multiplication of whole numbers to fractions. They interpret multiplication of a fraction by a whole number as repeated addition. They use the area model to understand multiplication of two fractions or mixed numbers. From diagrams and examples, the students observe that the “multiply across” rule for multiplication of fractions $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ is a sensible procedure.

This lesson falls near the end of a lesson cluster that focuses on fraction concepts and operations. In earlier lessons, students explored the meaning of parts and wholes; used visual models and numerical strategies to convert between fractions, decimals, and percents; compared and located rational numbers on a number line; and developed understanding of fraction addition and subtraction. In the lessons ahead, the meaning of fraction division will be explored and students will practice operations for fluency.

Math Goals (Standards for posting in **bold**)	• Extend concepts of whole number multiplication to fraction multiplication. (**Gr2 NS3.1**; Gr7 NS1.2; Gr7 MR2.2; Gr7 MR3.2) • Develop fraction multiplication concepts and procedures. (Gr5 NS2.0; Gr6 NS2.1; **Gr6 NS2.2**)
Summative Assessment	Future Week • Week 17: Fractions: Multiplication and Division (Gr6 NS2.1; Gr7 NS 1.2)

12.2 Fractions: Multiplication 1

PLANNING INFORMATION

Estimated Time: 45 – 60 Minutes

Student Pages * SP11: Ready, Set, Go * SP12: Fraction Multiplication: Repeated Addition * SP13: Fraction Multiplication: Area Model * SP14: Fraction Multiplication: Area Model (continued) * SP15: More Fraction Multiplication SP16: Fraction Multiplication Practice	**Materials**	**Reproducibles**
Homework SP16: Fraction Multiplication Practice	**Prepare Ahead**	**Management Reminders**
Assessment * SP25: Knowledge Check 12 R52-53: Knowledge Challenge 12 A54: Weekly Quiz 12	**Strategies for English Learners** Write vocabulary words on the board along with pictorial examples. proper fraction $\frac{3}{4}$ improper fraction $\frac{7}{4}$ mixed number $5\frac{1}{2}$ Compare conversational and academic meanings of proper, improper, and mixed.	**Strategies for Special Learners** Link multiplication of fractions to addition and multiplication of whole numbers. This will increase concept development and reduce the number of procedures that students need to remember.

* Recommended transparencies: See overheads 130-134 and 137 in the Teacher Resource Binder.

THE WORD BANK	
area model	The area model for multiplication is a pictorial way of representing multiplication. In the area model, the length and width of a rectangle represent factors, and the area of the rectangle represents their product.
proper fraction	A proper fraction is a fraction of the form $\frac{m}{n}$, where $1 \le m < n$. Example: The fractions $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{5}{6}$ are proper fractions.
improper fraction	An improper fraction is a fraction of the form $\frac{m}{n}$ where $m \ge n > 0$. Example: The fractions $\frac{3}{2}$, $\frac{17}{4}$, and $\frac{32}{16}$ are improper fractions.
mixed number	A mixed number is an expression of the form $p\frac{r}{q}$ for the number $p+\frac{r}{q}$, where $p \ge 1$ and $\frac{r}{q}$ is a proper fraction. We may regard $p\frac{r}{q}$ as an abbreviation for $p+\frac{r}{q}$. Example: The improper fraction $\frac{17}{4} = 4+\frac{1}{4}$ can be represented by the mixed number $4\frac{1}{4}$ ("four and one fourth"). This should not be confused with the product $4 \bullet \frac{1}{4} = 1$.
multiplicative identity property	The multiplicative identity property states that $a \bullet 1 = 1 \bullet a = a$ for all numbers a. In other words, 1 is a multiplicative identity. The multiplicative identity property is sometimes called the multiplication property of 1. Example: $4 \bullet 1 = 4$, $1 \bullet (-5) = -5$
multiplicative inverse	For $b \ne 0$, the multiplicative inverse of b is the number, denoted by $\frac{1}{b}$, that satisfies $b \bullet \left(\frac{1}{b}\right) = 1$. The multiplicative inverse of b is also referred to as the reciprocal of b. Example: The multiplicative inverse of 4 is $\frac{1}{4}$, since $4 \bullet \frac{1}{4} = 1$.

MATH BACKGROUND

Extending Multiplication to Fractions

Math Background 1

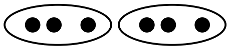
Preview/Warmup
Introduce

Two important mathematical ideas that were developed in previous lessons are used here to explain multiplication of fractions.

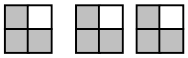
1. Multiplication by a whole number can be interpreted as repeated addition.

For whole numbers, this means that 2 groups of 3 can be written as $2 \times 3 = 3 + 3 = 6$. When extended to fractions, this means that 3 groups of $\frac{3}{4}$ can be written as

$$3 \times \frac{3}{4} = \frac{3}{4} + \frac{3}{4} + \frac{3}{4} = \frac{9}{4} = 2\frac{1}{4}.$$



2 groups of 3

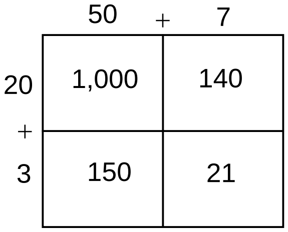


3 groups of $\frac{3}{4}$

2. An area model can be used to explain multiplication because the product of the base and height of a rectangle equals the area of the rectangle.

- When the base and height of the rectangle (i.e. factors) are written in an expanded form, the smaller rectangles inside of the large rectangle represent partial products. The sum of these partial products is the product of the factors. This diagram shows that

$$23 \times 57 = 1{,}000 + 150 + 140 + 21 = 1{,}311.$$

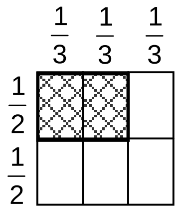


$$\begin{array}{r} 23 \\ \underline{\times \;\; 57} \\ 1{,}000 \\ 150 \\ 140 \\ \underline{+ \;\; 21} \\ 1{,}311 \end{array}$$

- The reasonableness of the multiplication rule for fractions is evident in products displayed with an area model. This diagram shows that

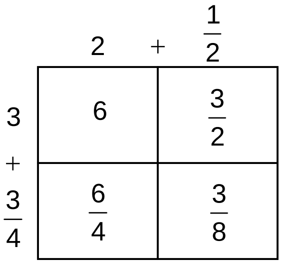
$$\frac{1}{2} \bullet \frac{2}{3} = \frac{2}{6}$$

The numerator of the product represents the number of parts in the shaded region (2). The denominator of the product represents the number of parts in the whole square (6).



- Multiplication of mixed numbers can be illustrated using an area model as well. This approach applies the distributive property to an expanded from of fractions as shown here:

$$2\frac{1}{2} \times 3\frac{3}{4} = \left(2 + \frac{1}{2}\right)\left(3 + \frac{3}{4}\right) = 6 + \frac{3}{2} + \frac{6}{4} + \frac{3}{8} = 9\frac{3}{8}$$



Area model rectangles are not drawn to scale.

MATH BACKGROUND (continued)

The Multiply Across Rule for the Multiplication of Fractions

Math Background 2

Teacher Mathematical Insight

The plausibility of the multiply across rule for multiplication of fractions has been established using the area model (see Math Background 1). Here we provide a mathematical justification of the multiply across rule on the basis of definitions and properties of arithmetic. As a warmup, we justify two basic identities.

<u>First identity (the big one identity)</u>: $\frac{a}{a} = 1$ for $a \neq 0$.

By the definition of fraction notation, $\frac{a}{b} = a \bullet \frac{1}{b}$, where $\frac{1}{b}$ is the multiplicative inverse of *b*.

Hence, $\frac{a}{a} = a \bullet \frac{1}{a} = 1$.

<u>Second identity (multiplicative inverse of a product)</u>: $\frac{1}{b \bullet d} = \frac{1}{b} \bullet \frac{1}{d}$ for $b,\ d \neq 0$.

To see that $\frac{1}{b} \bullet \frac{1}{d}$ is the multiplicative inverse of $b \bullet d$, we simply multiply it by $b \bullet d$:

$$(b \bullet d) \bullet \left(\frac{1}{b} \bullet \frac{1}{d}\right) = b \bullet \frac{1}{b} \bullet d \bullet \frac{1}{d} = 1 \bullet 1 = 1.$$

Here we have used the commutative property of multiplication, the multiplicative identity property, and the definition of $\frac{1}{e}$ as the multiplicative inverse of *e*.

<u>Multiply across rule</u>: $\frac{a}{b} \bullet \frac{c}{d} = \frac{a \bullet c}{b \bullet d}$ for $b,\ d \neq 0$.

Note that the second identity above is a special case of the multiply across rule, where the numerators are both equal to 1. For the general case, we use the associative and commutative properties of multiplication and the definitions to obtain:

$$\begin{aligned}
\left(\frac{a}{b}\right) \bullet \left(\frac{c}{d}\right) &= a \bullet \left(\frac{1}{b}\right) \bullet c \bullet \left(\frac{1}{d}\right) && \text{(definition of } \tfrac{a}{b} \text{ and } \tfrac{c}{d}\text{)} \\
&= a \bullet c \bullet \left(\frac{1}{b}\right) \bullet \left(\frac{1}{d}\right) && \text{(commutativity)} \\
&= a \bullet c \bullet \frac{1}{b \bullet d} && \text{(formula two proved above)} \\
&= \frac{a \bullet c}{b \bullet d} && \text{(definition of fraction notation)}
\end{aligned}$$

MATH BACKGROUND (continued)

The Multiplicative Inverse of a Fraction

Math Background 3

Teacher Mathematical Insight

It is intuitively clear that the multiplicative inverse of $\frac{a}{b}$ is $\frac{b}{a}$. This can be justified easily by using the multiply across rule, commutativity, and the big one identity.

$$\frac{a}{b} \bullet \frac{b}{a} = \frac{a \bullet b}{b \bullet a} = \frac{a \bullet b}{a \bullet b} = 1$$

Another way to express this, using the notation $\div$ for division, is that the multiplicative inverse of $a \div b$ is $b \div a$. In other words, $\frac{1}{a \div b} = b \div a$.

Preview of the Multiply-by-the-Reciprocal Rule for Division

Math Background 4

Teacher Mathematical Insight

The multiply-by-the-reciprocal rule for division is that

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \bullet \frac{d}{c} \quad \text{for} \quad b,\ c,\ d \neq 0.$$

This rule follows immediately from the rule for division (division by $\frac{c}{d}$ is defined to be multiplication by the multiplicative inverse of $\frac{c}{d}$), and the fact that the multiplicative inverse of $\frac{c}{d}$ is $\frac{d}{c}$ (proved in math background 3).

We will return to the multiply-by-the-reciprocal rule in 13.2 when we take up division of fractions.

TEACHING TIPS

Group Work: Questioning, Assessing, Accountability

Teaching Tip 1

Explore

Group work offers many benefits to students. Group work requires active participation (written and verbal) among learners, which increases the chance that more students will learn more mathematics. English language learners are more likely to participate in group discussions than whole class discussions because they are less intimidating.

Some questions and statements that promote thinking and keep students on task are:
- “I notice that Jose wrote ___ while Jesse wrote ____.” Are they both correct?
- “Beth, explain to Wanda why you wrote that answer.”
- “You all have the same answers. If I called on your group to explain how you got this, are you sure you would be able to convince the rest of the class that you are right?”
- “It appears that Terry has been doing the majority of the work in this group. The rest of you will be the ones to come to the overhead to explain this work in 5 to 10 minutes.”

12.2 Fractions: Multiplication 1

PREVIEW / WARMUP

Whole Class

- ➢ SP11*
  Ready, Set, Go

Math Background 1

- Introduce the goals and standards of the lesson. Discuss important vocabulary as relevant.
- Students practice the previously learned skills of whole number multiplication; represented as repeated addition and with an area model; and fraction addition. All three are important sub-skills for this lesson. Discuss as needed.

INTRODUCE

Whole Class

- ➢ SP12*
  Fraction Multiplication: Repeated Addition
- ➢ SP13*
  Fraction Multiplication: Area Model
- ➢ SP14*
  Fraction Multiplication: Area Model (continued)
- ➢ SP15*
  More Fraction Multiplication (start)

Math Background 1

- Guide students through the various exercises designed to build understanding of fraction multiplication.

  Looking at problems #1-7 on SP12, how is multiplying a whole number by a proper fraction like whole number multiplication? They can both be performed as repeated addition.

  Looking at problems #1-7 on SP13-14, how is multiplying a proper fraction by a proper fraction like whole number multiplication? The dimensions of the rectangles represent the factors. For proper fractions, the product of the numerators represents the number of parts in the shaded region (area), and the product of the denominators represents the number of parts in one whole square unit. This diagram suggests the plausibility of the multiply across rule $\left(\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}\right)$, seen in problem #8.

  Looking at problems #9-10 on SP14 and problems #1-3 on SP15, how can the area model be used to multiply a whole number by a mixed number? Write the mixed number in an expanded form. Then use the distributive property (write numerically or display visually with an area diagram) to find the partial products, and add them together. Repeated addition can be used to find the partial product of the whole number and fraction.

$$(3)\left(2\frac{1}{2}\right) = (3)\left(2 + \frac{1}{2}\right) = 3(2) + 3\left(\frac{1}{2}\right) = 6 + \frac{3}{2} = 7\frac{1}{2}$$

EXPLORE

Small Groups/Pairs

- ➢ SP15*
  More Fraction Multiplication (finish)

Teaching Tip 1

- Students do the next problems on the page as recommended. Invite students to come to the overhead to demonstrate their understanding of these methods for multiplying mixed numbers.

SUMMARIZE

Whole Class

- SP15*
  More Fraction
  Multiplication (finish)

- Invite students to the overhead to share alternative strategies for multiplying fractions.

  For problems 4 and 5, what are some ways to multiply two mixed numbers? (1) Make an area diagram and write the numbers in an expanded form. The partial products will include the product of a whole number, the product of a whole number and a fraction, and the product of two fractions. Then, add the partial products together. (2) Change both mixed numbers to improper fractions and use the rule illustrated in problem #8 on SP14.

PRACTICE

Individuals

- SP16
  Fraction
  Multiplication
  Practice

- This page is appropriate for practice as classwork or homework.

CLOSURE

Whole Class

- SP11*
  Ready, Set

- Review the goals and standards for the lesson.

12.2 Fractions: Multiplication 1

SELECTED SOLUTIONS	
SP11 Warmup (Go)	1. $5 + 5 + 5 + 5 + 5 + 5 = 30$ 2. $7 + 7 + 7 + 7 = 28$ 3. $\frac{5}{2} = 2\frac{1}{2}$ 4. $3\frac{3}{4}$ 5. $2{,}100 + 560 + 60 + 16 = 2{,}736$
SP12 Fraction Multiplication: Repeated Addition	1. $8 + 8 + 8 = 24$ 2. $\frac{2}{5} + \frac{2}{5} = \frac{4}{5}$ 3. $\frac{2}{7} + \frac{2}{7} + \frac{2}{7} = \frac{6}{7}$ 4. $\frac{2}{3} + \frac{2}{3} + \frac{2}{3} + \frac{2}{3} = \frac{8}{3} = 2\frac{2}{3}$ 5. $\frac{5}{6}$ 6. $\frac{8}{4} = 2$ 7. $\frac{12}{3} = 4$
SP13 Fraction Multiplication: Area Model	1. $\frac{1}{4}$ 2. $\frac{1}{8}$ 3. $\frac{3}{16}$ 4. $\frac{6}{12} = \frac{1}{2}$
SP14 Fraction Multiplication: Area Model (continued)	5. $\frac{1}{12}$ 6. $\frac{2}{6} = \frac{1}{3}$ 7. $\frac{4}{15}$ 8. $\frac{a}{b} \bullet \frac{c}{d} = \frac{ac}{bd}$ The numerator of the product represents the number of parts in the shaded region. The denominator of the product represents the number of parts in the whole square. 9. $100 + 30 = 130$ 10. $6 + 1\frac{1}{2} = 7\frac{1}{2}$
SP15 More Fraction Multiplication	1. $17\frac{1}{2}$ 2. $6\frac{6}{8} = 6\frac{3}{4}$ 3. $40\frac{4}{5}$ 4. $6 + 1\frac{1}{2} + 1\frac{1}{2} + \frac{3}{8} = 9\frac{3}{8}$ 5. $\frac{5}{2} \times \frac{15}{4} = \frac{75}{8} = 9\frac{3}{8}$
SP16 Fraction Multiplication Practice	1. $\frac{12}{7} = 1\frac{5}{7}$ 2. $23\frac{1}{3}$ 3. $\frac{2}{15}$ 4. $\frac{15}{45} = \frac{1}{3}$ 5. 4 6. $5\frac{1}{16}$

FRACTIONS: MULTIPLICATION 2

In this lesson, students practice concepts and skills presented in the previous fraction multiplication lesson, and they become more proficient in using fraction multiplication procedures.

This lesson falls near the end of a lesson cluster that focus on fraction concepts and operations. In earlier lessons, students explored the meaning of parts and wholes; used visual models and numerical strategies to convert between fractions, decimals, and percents; compared and located rational numbers on a number line; and developed understanding of fraction addition and subtraction. In the lessons ahead, the meaning of fraction division will be explored, and students will practice operations for fluency.

This week, the highlighted review topic is factorization. Exercises include the final concentrated practice on this topic. A summative assessment of the topic can be found in part two of the weekly quiz (part one is a formative assessment of the week). For students who do not demonstrate mastery at this time, a highlighted review practice worksheet (see Reproducibles) and a second form of the assessment (see Assessments) are available in the Teacher Resource Binder. The binder also includes a reproducible knowledge challenge for more proficient students and for enrichment.

Math Goals (Standards for posting in **bold**)	- Develop fraction multiplication concepts. (Gr2 NS3.1; **Gr6 NS2.2**) - Practice fraction multiplication procedures. (Gr5 NS2.0; Gr6 NS2.1; **Gr7 NS1.2**) - Review math concepts in prior lessons. - Demonstrate competency in factorization (highlighted review). (Gr4 NS4.1; Gr4 NS4.2; Gr5 NS1.4)
Summative Assessment	This week - Week 12: Factorization (Gr4 NS4.1; Gr4 NS4.2; Gr5 NS1.4) Future week - Week 17: Fractions: Multiplication and Division (Gr6 NS2.1; Gr7 NS 1.2)

12.3 Fractions: Multiplication 2

PLANNING INFORMATION

<table>
<tr><th colspan="3">Estimated Time: 30 – 45 minutes</th></tr>
<tr>
<td>Student Pages
SP0: Focus on Vocabulary 12
* SP17-18: Skill Builder 1
SP19: Skill Builder 2
SP20-21: Skill Builder 3
SP22: Test Preparation 12
* SP25: Knowledge Check 12</td>
<td>Materials</td>
<td>Reproducibles
* R51: Highlighted Review Practice 12
R52-53: Knowledge Challenge 12</td>
</tr>
<tr>
<td>Homework
SP0: Focus on Vocabulary 12
SP19: Skill Builder 2
SP20-21: Skill Builder 3
SP22: Test Preparation 12</td>
<td>Prepare Ahead</td>
<td>Management Reminders</td>
</tr>
<tr>
<td>Assessment
* SP25: Knowledge Check 12
R52-53: Knowledge Challenge 12
A54: Weekly Quiz 12
A55-56: Highlighted Review Quiz 12</td>
<td>Strategies for English Learners</td>
<td>Strategies for Special Learners
Link multiplication of fractions to addition and multiplication of whole numbers. This will increase concept development and reduce the number of procedures that students need to remember.</td>
</tr>
</table>

* Recommended transparencies: See overheads 135-138 in the Teacher Resource Binder.

12.3 Fractions: Multiplication 2

MATH BACKGROUND

Using the Multiply Across Rule Efficiently

Math Background 1

Introduce

In a previous lesson, students were introduced to the multiply across rule.

$$\frac{a}{b}\times\frac{c}{d} = \frac{ac}{bd}\,.$$

Though this rule is helpful, it sometimes yields a fraction that is not in lowest terms and may be difficult to simplify. For example:

$$\frac{7}{27}\times\frac{9}{28} = \frac{63}{756}$$

Using arithmetic and equality rules, the process can be made more efficient by simplifying fractions prior to multiplying.

Equivalent Expressions	Reasons
$\frac{7}{27}\times\frac{9}{28}$ w	given expression
$= \frac{7\bullet 9}{27\bullet 28}$	multiply across rule
$= \frac{9\bullet 7}{27\bullet 28}$	commutative property of multiplication
$= \frac{9}{27}\times\frac{7}{28}$	multiply across rule
$= \frac{1}{3}\times\frac{1}{4}$	replace each factor with simplest equivalent fraction
$= \frac{1\bullet 1}{3\bullet 4}$	multiply across rule
$= \frac{1}{12}$	multiplication

In practice, this procedure typically looks like this:

$$\frac{\overset{1}{\cancel{7}}}{\underset{3}{\cancel{27}}}\times\frac{\overset{1}{\cancel{9}}}{\underset{4}{\cancel{28}}} = \frac{1\bullet 1}{3\bullet 4} = \frac{1}{12}$$

12.3 Fractions: Multiplication 2

INTRODUCE

Whole Class

- SP17-18*
  Skill Builder 1

Math Background 1

- Use problems #1-9 to review previously introduced concepts and skills used in fraction multiplication. As students become more proficient in using the multiply across rule, encourage them to simplify expressions prior to multiplying to make using the rule more efficient.

PRACTICE

Individuals

- SP0
  Focus on Vocabulary 12
- SP19-21
  Skill Builders 2-3
- SP22
  Test Preparation 12
- R52-53
  Knowledge Challenge 12

- These student pages are appropriate for practice as class work or homework leading towards basic competency in the topic.
- The Teacher Resource Binder includes challenge problems for more proficient students and for enrichment.

ASSESSMENTS

Individuals

- A54
  Weekly Quiz 12
- SP25*
  Knowledge Check 12
- A55-56
  Highlighted Review Quiz 12
- R51*
  Highlighted Review Practice 12

- Use the knowledge check to review and practice topics for the week. The weekly quiz is a formative assessment.
- Part A of the highlighted review quiz is a summative assessment of the highlighted review. If students have not demonstrated basic competency, provide extra practice using the highlighted review reproducible, and reassess with part B of the highlighted review quiz.

12.3 Fractions: Multiplication 2

<table>
<tr><th colspan="4">SELECTED SOLUTIONS</th></tr>
<tr><td>SP0
Focus on Vocabulary 12</td><td>1. f, i
2. d, m
3. f, i
4. f, i</td><td>5. a, c
6. b, k
7. c
8. e, h</td><td>9. h
10. e
11. g</td></tr>
<tr><td>SP17
Skill Builder 1A</td><td>1. $\frac{3}{2}$ or $1\frac{1}{2}$
2. $\frac{3}{2}$ or $1\frac{1}{2}$
3. $\frac{3}{2}$ or $1\frac{1}{2}$</td><td>4. $\frac{15}{4}$ or $3\frac{3}{4}$
5. $\frac{15}{4}$ or $3\frac{3}{4}$
6. $\frac{15}{4}$ or $3\frac{3}{4}$</td><td>7. $\frac{1}{2}$
8. $\frac{1}{2}$
9. $\frac{1}{2}$</td></tr>
<tr><td>SP18
Skill Builder 1B</td><td>10. $\frac{5}{2}$ or $2\frac{1}{2}$
11. $\frac{1}{4}$
12. $18\frac{2}{3}$</td><td>13. $\frac{8}{27}$
14. $\frac{4}{3}$ or $1\frac{1}{3}$
15. $\frac{1}{12}$</td><td>16. $\frac{5}{2}$ or $2\frac{1}{2}$
17. 28
18. $\frac{4}{5}$</td></tr>
<tr><td>SP19
Skill Builder 2</td><td>1. 5
2. $\frac{3}{10}$
3. $1\frac{2}{3}$
4. A number greater than one that has exactly 2 factors, one and itself.</td><td>5. A number greater than one that has more than 2 factors.
6. 2
7. 2, 3, 5, 7, 11, 13, 17, 19, 23, 29</td><td>8. 0, 2, 4, 6, 8
9. 7, 2
10. 24, 16, 39-
11. $\frac{10}{100}$ = 0.10 = 10%
12. $\frac{75}{100}$ = 0.75 = 75%</td></tr>
<tr><td>SP20
Skill Builder 3A</td><td>1. $4\frac{4}{9}$
2. $\frac{3}{20}$
3. $3\frac{3}{4}$</td><td>4. 3; 27; 3,063
5. 453; 3,064; 72
6. 1, 31
7. Prime
8. 2.6</td><td>9. 12.27
10. $13\frac{1}{8}$
11. $-\frac{5}{9}$</td></tr>
<tr><td>SP21
Skill Builder 3B</td><td>12. 25,000
13. 200,000
14. 6,000
15. 250,000
16. -52
17. $-2\frac{2}{3}$</td><td colspan="2">18.
<table>
<tr><td>dimes (x)</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td><td>x</td></tr>
<tr><td>nickels (y)</td><td>0</td><td>2</td><td>4</td><td>6</td><td>8</td><td>10</td><td>$2x$</td></tr>
</table>
Rule: $y = 2x$</td></tr>
<tr><td>SP22
Test Preparation 12</td><td>1. A
2. H</td><td>3. A
4. H</td><td>5. C
6. F</td></tr>
</table>

12.3 Fractions: Multiplication 2

<table>
<tr>
<td>SP25
Knowledge Check 12</td>
<td>1.<table><tr><th>input (<i>x</i>)</th><th>output (<i>y</i>)</th></tr><tr><td>3</td><td>14</td></tr><tr><td>10</td><td>42</td></tr><tr><td>0</td><td>2</td></tr><tr><td>11</td><td>46</td></tr><tr><td>20</td><td>82</td></tr></table></td>
<td>2. $y = 4x + 2$
3. $\frac{3}{5} + \frac{3}{5} + \frac{3}{5} + \frac{3}{5} = \frac{12}{5} = 2\frac{2}{5}$
4. 1
5. $\frac{13}{3}$ or $4\frac{1}{3}$</td>
<td>6. $\frac{1}{15}$
7. 61, 71
8. 39, 49, 69</td>
</tr>
<tr>
<td>SP26
Home-School Connection 12</td>
<td>1. $y = 25x + 15$</td>
<td>2. $\frac{3}{5} + \frac{3}{5} = \frac{6}{5} = 1\frac{1}{5}$</td>
<td>3. 4</td>
</tr>
<tr>
<td>A54
Weekly Quiz 12</td>
<td>1.<table><tr><th>Input (<i>x</i>)</th><th>output (<i>y</i>)</th></tr><tr><td>4</td><td>6</td></tr><tr><td>0</td><td>3</td></tr><tr><td>20</td><td>18</td></tr><tr><td>8</td><td>9</td></tr><tr><td>16</td><td>15</td></tr><tr><td>12</td><td>12</td></tr><tr><td>x</td><td>$\frac{3}{4}x + 3$</td></tr></table></td>
<td>2. $y = \frac{3}{4}x + 3$
3. $2\frac{2}{5}$ or $\frac{12}{5}$
4. 2</td>
<td>5. $\frac{8}{9}$
6. $\frac{1}{7}$
7. $\frac{15}{4}$ or $3\frac{3}{4}$</td>
</tr>
<tr>
<td>A55
Highlighted Review Quiz 12A</td>
<td>1. $18 = 2 \bullet 3 \bullet 3$
2. $72 = 2 \bullet 2 \bullet 2 \bullet 3 \bullet 3$
3. $500 = 2 \bullet 2 \bullet 5 \bullet 5 \bullet 5$</td>
<td>4. 315
5. 7, 2
6. 24, 16, 39</td>
<td>7. 1, 2, 3, 4, 6, 9, 12, 18, 36
8. C</td>
</tr>
<tr>
<td>A56
Highlighted Review Quiz 12B</td>
<td>1. $24 = 2 \bullet 2 \bullet 2 \bullet 3$
2. $88 = 2 \bullet 2 \bullet 2 \bullet 11$
3. $400 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 5 \bullet 5$</td>
<td>4. 24
5. 11, 13
6. 12, 21</td>
<td>7. 1, 2, 4, 8, 16, 32
8. B</td>
</tr>
<tr>
<td>R51
Highlighted Review Practice 12</td>
<td>1. $30 = 2 \bullet 3 \bullet 5$
2. $64 = 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2 \bullet 2$
3. $450 = 2 \bullet 3 \bullet 3 \bullet 5 \bullet 5$</td>
<td>4. 350
5. 5, 53
6. 26, 18, 27</td>
<td>7. 1, 2, 4, 13, 26, 52
8. D</td>
</tr>
<tr>
<td>R52-53
Knowledge Challenge 12</td>
<td colspan="3">1. $12 + n < 30$
2. $4 \times 6 = 3 \times n$
3. The actual cost is $8.20. He is correct that he has enough money, but uses the wrong estimates. He should round up instead of down to ensure he has sufficient funds.
4. 30 inches
5. $\frac{7}{3}$ or $2\frac{1}{3}$ yards
6. 214.4 ounces
7. $x = 4$
8. 21
9. 1, 2, 3, 4, 6, 9, 12, 18, 36
10. 1, 3, 14</td>
</tr>
</table>