USING *INTRODUCTION TO ALGEBRA* WITH ENGLISH LEARNERS

By Leslie Garrison

PART 1: BASIC CONCEPTS
THE CHALLENGE

Mathematics teachers face two challenges when working with linguistically diverse classrooms: How to address the language diversity of their students, and how to address the diversity in mathematical understanding (Garrison, 2006). When supporting English learners, teachers must try to provide all the support students need to understand the language and the mathematics they are presenting, but not so much support that it interferes with their learning.

Conversational vs. Academic Language
Teachers can easily misdiagnose the language proficiency of students. This is because conversational English, the language spoken among friends and in non-instructional settings, is what most students learn first. Academic language, consisting of technical vocabulary and complex reasoning, is critical in solving mathematics problems, but develops much more slowly.

This situation can be illustrated using the following problem taken from the released items from the California High School Exit Exam.

Sally puts $200.00 in a bank account. Each year the account earns 8% simple interest. How much interest will be earned in 3 years?

If you were to present this problem to students, including English learners, they may respond by saying “I don’t get it.” If asked if they understand the words, they may respond “Yes,” because they are familiar with the words in conversational vocabulary. However, students may be misinterpreting the words in this mathematical context. For example, an “account” is a “story” and “simple interest” might be interpreted as “basically interesting”. Without building additional vocabulary within the context of the banking system, the student might never understand the problem as it was written.

The issue here is not whether students should be able to correctly understand and solve the above problem as originally written. The issue is how to make mathematics problems accessible to all students so they have the opportunity to learn both the language and the mathematics. This can be accomplished by teaching the vocabulary and developing the context.

Vocabulary Instruction

The key to learning new vocabulary is to 'hook it' to something the student already knows. In the case of English learners, their primary language can often provide a link. Words in two different languages that are similar are called cognates. For example, la suma, in Spanish and the English word sum are cognates because they are similar and have the same meaning.

Other linkages can be made between visual representations and word meanings. These can be helpful for English learners as they reduce their dependence on language for understanding. For example, circle, diameter, trapezoid, and perimeter, can easily be represented visually. Other terms, such as numerator, denominator, subtrahend, and inequality can be shown through examples where the key component is highlighted. Vocabulary words such as prime number, improper fraction, and even and odd can be depicted by an example and a very brief explanation of the key characteristics.

Beginning speakers of a language frequently categorize unfamiliar words by sound, while more experienced speakers organize words by categories. For example, beginning speakers of English might mix up the words kitchen and chicken because they both sound similar. Fluent speakers will have these word in distinct categories based on their characteristics and not find them at all confusing. This concept is important for mathematics teachers because many words used in mathematics and general conversation have multiple meanings and can be very confusing to English learners. These multiple meanings often go unnoticed by fluent speakers. Some examples of perplexing words include sum vs. some, difference vs. different, hours vs. ours, table (chart) vs. table (furniture), product (for multiplication) vs. product (to be purchased), round (estimate) vs. round (circular).

When introducing a word that might have another meaning, discuss the meaning and spelling for both words and help English learners understand when each word would be used. If the additional meaning is not as common as the one you are using, then there is no need to include it in the current lesson. When introducing new words, use multiple modalities as they help students retain the vocabulary. Specifically, introduce the word, spelling, picture, example, personal definitions, class definitions, and/or formal definitions. Use the new words frequently throughout the lesson and surround the word with enough context so that its meaning is continually reinforced.

Complex Definitions

When the formal definition is important but difficult to understand, teach the lesson differently. This can be done by starting with the context and concept. Once those are understood, move to the term and definition. An example may be the associative, commutative, and distributive properties. English learners will be more able to understand the properties than the formal definitions, so the properties are the best place to start. The lesson could be approached by:

1) introducing a sample problem which demonstrates the property,
2) discussing the property in class and determining when it applies,
3) providing and having students create other examples,
4) creating a class definition of the property,
5) supplying the formal name and reading the formal property definition, and
6) comparing and contrasting the formal and class definitions.

Since so much of Algebra is dependent on these concepts, the extra time spent can be well worth the effort.

Opportunities to Practice
Repeated and dispersed opportunities to practice new vocabulary is embedded into the *Introduction to Algebra* curriculum. This is a vital link for English learners who are trying to expand their vocabulary. Make sure to capitalize upon these chances to reinforce vocabulary with English learners.

Most students, including native speakers, can benefit from creating and using vocabulary flash cards. For beginning speakers these can have the vocabulary word on one side and a picture description on the other. More advanced speakers can use the word and written definition. Students may have their own set of flashcards, or the teacher can have a set for use with the overhead. As a class warm up, or in the last few minutes before the end of class, the words and definitions can be quickly reviewed. As words and definitions are learned, replace them with new ones.

PROBLEM SELECTION AND MODIFICATION

Tips for Level I English Learners – Focus on Listening
Level I English learners are generally recent immigrants who have not previously studied English. There can be a wide variance in their mathematics preparation and while some will struggle with mathematical concepts, others will be among the most accomplished mathematics students in your class. This group is made up of students who have had the opportunity to learn rigorous mathematics before coming to the United States. They will be well versed in the mathematics and just need to understand what the problem is requiring in order to excel.

In general, while the teacher continues to build on their English vocabulary, provide visual cues to help students understand the problem and allow them to demonstrate what they know.

Table 1: **Some Strategies that Develop Listening, Speaking, Reading, Writing, and Vocabulary Skills in A Linguistically Diverse Mathematics Classroom**

	LEVEL I	**LEVEL II**	**LEVEL III**	**LEVEL IV**
General Description	Newcomer with little or no knowledge of English. Pre-production or silent phase.	Understands basic conversational English but lacks academic English skills.	Engages in conversational English but needs support in reading and writing.	Has command of basic English. Needs support in written expression.
Listening (teacher)	Provide extensive visual cues such as diagrams with verbal and written explanations.	Include vocabulary explanations using simplified language allowing time to process what is said and done.	Pre-teach vocabulary and then use regularly throughout lesson.	Use grade and subject appropriate vocabulary. Pre-view/review new words as necessary.
Speaking (students)	Be able to show rather than tell what they did.	Respond to structured questions. (i.e., What did you do first?)	Share their work using English in small group settings.	Verbally share thinking with the class.
Reading (teacher)	Rely on numeric representations and diagrams.	Pre-teach new words, or words used in a new context.	Pre-teach new vocabulary and read and discuss question together.	Read and discuss question together.
Writing (students)	Write numeric examples.	Write numeric example and simple phrases or sentences.	Write sentences bolstered with numeric examples.	Write complete and descriptive sentences.
Vocabulary (teacher)	Pre-teach vocabulary using visual representations for new words.	Pre-teach vocabulary with visual or numeric examples.	Provide examples and non-examples, have the student infer and write the definition.	Provide written definitions and have student supply numeric examples.

Tips for Level II English Learners – Focus on Speaking

Level II English learners will know conversational English but often lack the academic vocabulary to fully understand complex directions and explanations in mathematics. They may have some of the characteristics of the type of student discussed above who understands the words used in the banking problem, but still doesn't understand its meaning.

In general, small group work is a good instructional technique for Level II students. They are provided an opportunity to check their understanding and get practice speaking English in a less public environment.

Tips for Level III English Learners – Focus on Reading

Level III English learners will generally be able to read and understand written problems and directions if they have vocabulary support. Preview the problems looking for new words or words used in a new context, and address their meaning either before students start or within the context of the problem. Use new vocabulary repeatedly within the lesson and require students to do the same. If these strategies have employed and the students are still having trouble, the root is likely mathematical rather than linguistic. One way to confirm this is to have the students explain the problem in their own words. From these explanations, the teacher can generally determine whether the student doesn't understand what the problem is asking (language issue) or doesn't know how to solve it (mathematics issue).

Tips for Level IV English Learners – Focus on Writing
Level IV English learners are nearly fluent in understanding and speaking English. While new vocabulary always remains a concern, the more academic aspects of writing English may be where assistance is needed. Discussing possible responses can support writing. This helps clarify ideas and identify key vocabulary words. Words to be used in definitions or explanations can be highlighted at this time or written on the board.

Avoid writing responses on the board and having students copy them down, even with Level I and Level II speakers. While allowing students to 'complete' the assignment, it does very little to help them learn the language. A grammatically correct but copied sentence gives no indication of the student's level of understanding of mathematics or English. A poorly constructed but original sentence usually provides a window into the student's thinking and indicates aspects of English the teacher may want to address. When correcting grammar and spelling in a student's mathematics, a good rule of thumb is to select one or two points to stress and work with them until they have been mastered. Too much red ink, especially for struggling writers, can make students very reluctant to put anything on paper.

Table 1 summarizes some strategies to develop listening, speaking, reading, writing, and vocabulary skills in a linguistically diverse math classroom. *Table 2* suggests ways that teachers and English learners can work together in a problem-solving environment.

Table 2: **Establishing a Problem Solving Environment in Linguistically Diverse Mathematics Classroom**

	LEVEL I	**LEVEL II**	**LEVEL III**	**LEVEL IV**
Teacher presents the problem	Act out or diagram the problem as you are speaking whenever possible.	Restrict or pre-teach vocabulary. Present verbally and in writing. Use graphic organizers.	Present verbally and in writing. Use graphic organizers.	Present problem and discuss.
Teacher checks for understanding	Will the answer be bigger, smaller, or between the other numbers in the problem?	What do you know? What are you trying to find out?	Please restate the problem in your own words.	What will be your strategy to solve this problem?
Students work in cooperative groups	Pair/group with student who speaks their language.	Place in small cooperative groups to develop verbal skills and understanding.	Place in groups that include strong English models.	Groups using mathematics rather than English as a criteria.
Students share their thinking	Show the solution steps.	Show what you did and provide phrases to explain why.	Write or tell solution steps and tell why they did each one.	Provide a written description of their thinking in solving the problem.

PART II: COMBINING MATHEMATICS AND LANGUAGE INSTRUCTION

(Adapted from: Garrison, L. (2006). *UnLATCHing Mathematics Instruction for English Learners*, NCSM Journal of Mathematics Education Leadership.)

Many mathematics teachers come to professional development sessions with a basic understanding of how to teach content. Increasingly, teachers are becoming aware about second language acquisition and the need to make connections between mathematics instruction and English learners. The issue at hand, then, is to see how language acquisition strategies can become an integral part of content instruction. The Language Acquisition through Content Hierarchy (LATCH) model was developed to assist teachers integrate content and language acquisition strategies and use them to differentiate instruction for English learners.

In multilingual classrooms, teachers of mathematics face two challenges: how to address the language diversity of their students, and how to address the diversity in mathematical understanding. This section addresses strategies to help teachers differentiate mathematics instruction for students with a range of mathematical and English proficiencies.

DIVERSITY IN THE CLASSROOM

Language Diversity

Part of the reason English learners struggle in mathematics is that rather than being language free, mathematics uses language that is a highly compressed form of communication where each word or symbol often represents an entire concept or idea. In a literature text, readers can comprehend a passage if they are familiar with 85% – 90% of the words. The other words and their meanings can often be gleaned through context. Mathematics problems, on the other hand, generally require the student to understand nearly every word as there is seldom enough context provided with the problem to assist with unfamiliar words or concepts. Another problem that English learners encounter is that sometimes they recognize a word, but the meaning they know for the word is different from the intended meaning and therefore does not help them understand the problem.

The Natural Language Approach to language acquisition (Krashen, 1983) states that the process of learning a second language often mirrors that used by a child to learn a primary language. Children first learn the names of common objects: items repeatedly introduced visually and physically. Learning through direct experiences with concrete examples provides a **context-embedded** environment in that the words and their meanings are supported by physical objects or are otherwise familiar to the child. If students don't have the vocabulary or experiential background to understand, then they need to be provided this information in a concrete manner that builds upon experiences that are familiar.

Assigning an unfamiliar problem without any linguistic support creates what Jim Cummins calls a **context-reduced** environment. In these cases, it is presumed the student has the experience and vocabulary necessary to understand the problem. Cummins highlighted the importance of context in comprehension when he described the Socio-Linguistic Approach

(1979) as including two sets of skills required for language proficiency. He calls the first set Basic Interpersonal Communication Skills (BICS). BICS refers to context-embedded communication that takes place in every day interactions between individuals. Greetings, discussions of the weather, relaying what just happened on the playground, are all examples of BICS.

The second set of language skills involves Cognitive Academic Language Proficiency (CALP). In the case of CALP, communication takes place in a context-reduced environment, or one in which cues, such as visuals, gestures, or a familiar topic are not present. The primary distinction between BICS and CALP rests in the extent to which the context is embedded in the communication.

Cognitive Academic Language Proficiency covers two broad areas: Cognitive proficiency and academic language. The former refers generally to mathematical reasoning including the "higher level of language development [that] includes comparing, classifying, inferring, problem solving, and evaluation" (Williams, 2001; p.2). The academic language, as it applies to a mathematics classroom, is a broad term that encompasses the skills needed to succeed in school such as reading, writing, and the language skills required to communicate the reasoning behind a mathematical solution. It also includes the technical and specialized vocabulary and terms used in mathematics classes (Chamot and O'Malley, 1994). These higher-order thinking and language skills are found in classrooms where the language is complex and the tasks are cognitively demanding (Collier, 1988; Egbert and Simich-Dudgeon, 2001). These environments can be very challenging for students who have yet to gain Cognitive Academic Language Proficiency.

Language minority students often appear to be English proficient and yet perform poorly in content areas because, while they have some proficiency in interpersonal or conversational English, they lack proficiency in the content specific vocabulary, which can inhibit the development of academic skills (Cummins, 1979). As a result, students who lack English skills often find themselves falling farther and farther behind in mathematics. Thus, teachers find themselves searching for a variety of instructional strategies that will enhance learning for students at every level of English as well as mathematical proficiency.

Diversity in Mathematical Understanding

Many classrooms have as much diversity in student understanding of mathematics content as they do in language proficiency. This disparity is perhaps greatest in mathematical problem-solving. This critical area is not only a major goal of mathematics; it is a major means of doing mathematics. In fact, mathematical problem-solving should play a central role in the learning of mathematics (Hiebert, et al. 1996; Heibert, et al. 1997).

George Polya (1957) published pioneering work in the area of problem solving with his book, "How to Solve It." He outlines four steps in problem solving in the text to include:
1) Understanding the Problem; 2) Devising a Plan; 3) Carrying out the Plan; and 4) Looking Back. For English learners the greatest challenge happens in the first step, as they will not be able to solve a problem they can't understand. Once the problem is understood, the second and most cognitively challenging step is devising a plan. Polya provides many suggestions on

how to help students devise their own plan as he feels the plan must be their own if they are to learn problem solving. The following section looks at ways to help students, with varying mathematical skills, devise and carry out a plan for problem solving.

There is general consensus among mathematics educators that when students engage in problem solving, they progress from concrete to more abstract representations as their understanding increases (Stevenson & Stigler, 1992; Marzano, 1998; Good & Brophy, 2003; Shapiro, 2004). The Principles and Standards for School Mathematics (2000) discusses this progression and stresses the importance of allowing students to construct conceptual knowledge by building upon what they already know. Prior experiences provide a concrete base from which new, often more abstract concepts, can be developed.

Carpenter, Fennema and Franke (1996) identified this concrete to abstract progression in mathematics in their work with students. They found that when given a problem to solve, students use a variety of strategies. Some students use more concrete strategies such as direct modeling, drawing a picture or diagram, or using simpler numbers, while others are able to use algorithms, variables, and write equations. The strategies that students employ depend on their understanding of the problem, the difficulty of the numbers, and the set of skills, understandings and prior knowledge they bring to the situation. In effect, as students gained more experience, direct modeling strategies give way to procedures utilizing more abstract thinking.

Bridging Language Acquisition and Mathematics Content
Cummins' work (1994) provides a framework for language acquisition and how it interfaces with content area instruction. He proposed what has come to be known as Cummins' Quadrants which graphically depict the four linguistic domains of English learners. An adaptation of these quadrants has been made for the LATCH model and appears in the inner rectangle of *Figure 1.*

According to Echevarria and Graves (2003), the "[vertical] continuum represents contextual support, ranging from contextually embedded communication, wherein meaning can be derived from a variety of clues such as gestures, visual clues, and feedback, to context-reduced communication, which relies primarily on linguistic messages or written texts, which give few, if any, contextual clues (p.43)." The [horizontal] continuum relates to the cognitive demands of the task. Since cognitive demand can have a different connotation in mathematics instruction, this axis may be thought of as concrete to abstract to match the sequence noted by Carpenter, Fennema and Franke (1996). The concrete end of the continuum includes solving a problem using manipulatives or drawings and is generally where the greatest number of students will have success. It is therefore the point of greatest access for students (Carpenter, Fennema, & Franke, 1996). Abstract solution strategies, such as writing equations or providing mathematical proofs, generally require the most previous knowledge and experience and therefore fewer students will be successful if they start there. In short, **for any given problem**, more students will be able to solve it initially using concrete strategies than abstract strategies.

The fourfold way (representing mathematical ideas using pictures, numbers, symbols, and words), used throughout the *Introduction to Algebra* materials, incorporates the concrete to abstract learning sequence directly into the curriculum. Initially, students explore problems using manipulatives and then draw a picture of what they built. These provide the foundation for more abstract representations of the problem, such a tables and graphs, which provide the context for symbolic notation. This sequence of skills matches the progression from concrete to abstract concepts important in developing a deep understanding of mathematics.

The value in Cummins' quadrants is their ability to link language acquisition issues to those of content instruction. The LATCH model provides more direction on how mathematics instruction can be adapted to meet the needs of English learners.

Defining the Domains within the Context of Mathematics Instruction
The LATCH model connects the Cummins' Quadrants (inner rectangle) to practical applications of presenting mathematics lessons to English learners (outer rectangle). Each of the quadrants in the inner rectangle describes a different type of student with specific learning needs:

Students in Quadrant A (upper left) are ones who struggle with both English and mathematics. They need strong linguistic support in order to understand the problem. The LATCH model supplies instructional suggestions (Pair with a partner, Act problem out, Provide visual cues. . .) that could assist in this process. The students in Quadrant A also need support in mathematics. The LATCH model shows that their solution strategies will likely be more concrete than others in the class. While it might not be the instructional goal for them to remain in this area, they will likely be more successful if they have the opportunity to initially use more concrete strategies such as using manipulatives, drawing a picture, or making a table.

Students in Quadrant B (upper right) lack proficiency in English, but have strong mathematical understanding. This mathematical background was probably built in their first language, as this is the quadrant where recent immigrants who have had strong mathematics instruction in their native tongue are located. Quadrant B students may need strong instructional supports to understand the problem, but once they do, they can use more sophisticated strategies to solve it, a key difference from students in Quadrant A.

Students in Quadrant C (lower left) have greater proficiency in English, and while they may still need support, it will more likely be in reading and writing. This group of students does need support in mathematics, however. They will be more successful with concrete problem solving methods such as drawing pictures and finding patterns as their initial strategy.

Students in Quadrant D (lower right) are the ones who need fewer supports in English and are able to make abstract associations in mathematics. They will likely understand the problem at hand and can solve the problem in the specific as well as the general case. They may only need linguistic support in writing the justification of their solution.

Figure 1: **Language Acquisition Through Content (Adapted from the Cummins' Quadrants and the LATCH model)**

Context Embedded Instructional Strategies

Classroom organization
- Pairs include a student who speaks the same language.
- Groups based on English skills rather than mathematical criteria.

Teacher's Actions
- Pre-teach vocabulary using visuals.
- Provide extensive cues such as diagrams.
- Rely on numeric representations and diagrams to bolster understanding.
- Structure questions for responses. (e.g. What did you do first?)
- Act problems out.

Students' Actions
- Students respond using simple written phrases or numeric examples.
- Allow student to show rather than tell about their work.

Concrete Mathematical Solution Strategies

- Solve using manipulatives
- Draw a picture
- Find a pattern
- Make a table

A • English proficiency: Level I, Level II • Math proficiency: Lower	**B** • English proficiency: Level I, Level II • Math proficiency: Higher
C • English proficiency: Level III, Level IV • Math proficiency: Lower	**D** • English proficiency: Level III, Level IV • Math proficiency: Higher

Abstract Mathematical Solution Strategies

- Solve with variables
- Write an equation
- Make generalizations
- Provide a mathematical proof

Context Reduced Instructional Strategies

Classroom organization
- Groups include a strong English speaker as a model.
- Groups are based on mathematical skills rather than English criteria.

Teacher's Actions
- Read and discuss the problem together.
- Use new vocabulary regularly throughout the lesson.
- Give examples and non-examples of new words to let students infer meaning.

Students' Actions
- Ask students to verbally share their thinking with the class.

How to Use the Language Acquisition through Content Hierarchy

The classroom teacher can use the model to differentiate instruction for a classroom of students who are diverse in both English language proficiency and mathematical skill (see *Figure 2* for an example). Some mathematical content strategies that students are likely to use when solving a problem are identified in the outer rectangle and link to the *x*-axis. Strategies to help students understand the problem are found in the outer rectangle and link to the language or *y*-axis. In general, students should be allowed to solve problems using the methods that make sense to them, but they should also be exposed to more sophisticated (or abstract) solution strategies so their thinking can advance. The fourfold way in *Introduction to Algebra* capitalizes on this notion by having students employ concrete strategies, such as modeling or pictures and then progressively moving them to more abstract strategies such as symbolic notation.

Figure 2: **Using the LATCH Model to Differentiate Instruction: An Example**

<table>
<tr><td colspan="2">The Problem: A farmer put all her ducks and sheep in a pen.
When she counted the heads, she tallied 20.
When she counted the feet, they added up to 54.
How many ducks and how many sheep did she have?</td></tr>
<tr><td>Differentiation for Quadrant A:

The teacher could help the beginning English speakers understand the problem by using pictures of the animals mentioned in the problem. Even the word pen can be misleading, as many English learners will think of a writing instrument. Pictures of an animal pen would also need to be included. A simplified version using three of each animal could be depicted visually and the students asked to determine the number of heads and feet shown in the picture. This would allow the students in A to visualize what the problem is asking, and to solve it initially by using a direct model strategy (counting actual heads and feet). From here, they could make their own drawings or charts to solve the problem with larger numbers.</td><td>Differentiation for Quadrant B:

Students could be introduced to the problem using the pictures as in A, and, once they understood what was being asked, could employ large numbers. Particularly adept students could be asked to make a table showing the results for all even numbers between 54 and 64 and asked to look for a pattern. They could demonstrate their thinking through a chart or equation.</td></tr>
<tr><td>Differentiation for Quadrant C:

Students would likely understand the problem, but be at a loss on how to solve it. It could be modified for this group by reducing the numbers to 8 heads and 24 feet. If they still have problems, they should be encouraged to solve the problem through direct modeling or drawing pictures.</td><td>Differentiation for Quadrant D:

These students will need little if any support in understanding the question. After they solve the problem as stated, they should be challenged to construct an equation that would always work, no matter how many sheep and ducks were in the pen.</td></tr>
</table>

An assumption in the content strand is that students working at a more abstract level can solve a problem using concrete methods as well. However, this is not always the case, especially among teachers who have not learned mathematics using the concrete models. For them, the sequence can be in reverse order. This brings up three important points:

1) When instruction fails to include the models that underlie a concept, students will not necessarily develop them on their own.
2) Teachers need to know and understand the concrete models that underlie concepts so they can help students use them to create conceptual understanding.
3) The opportunity to use non-linguistic representations (i.e., concrete representations) increases student achievement (Marzano, Gaddy, & Dean, 2000). Therefore they should be included in mathematical instruction.

The professional development of the LATCH model allows teachers to draw upon their previous knowledge of teaching and mathematics to develop a personal instrument for instructional differentiation. This provides teachers with a meaningful tool to use in instructional planning and a reminder of strategies at their disposal to meet the needs of all the students in the classroom. It can help answer the question heard by teachers across the nation, how can I teach mathematics to a student who is not fluent in English?

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